2025/2026學年學界數學比賽　高中組　　(**2026/01/25　14:30– 16:30**)
Concurso Interescolar de Matemática do ano lectivo de 2025/2026　Divisão Sênior
Macau Interschool Mathematics Competition in 2025/2026　Senior Division

學校　　姓名　　座號
School _____Name _____Seat No._____

禁止翻閱試卷

Don't open the script

- 比賽共有9題：題1 – 4是選擇題，每題5 分，把答案的英文字母或對應的答案按題號填寫在右方空格內。
  題5–9是證明題，每題25 分，請把詳細的解題過程寫在附上的A4紙上，並在左上角寫上題目編號及姓名。

- 可用鉛筆或原子筆作答，圓規及直尺作圖，禁止使用塗改液及塗改膠帶，量角器、任何類型之計算器或電子產品。

- There are 9 questions. Questions 1–4 are multiple-choice questions with 5 marks each. Please write the letter of correct answer in the blanks on the right.
  Question 5–9 are proof-related with 25 marks each. Please write the arguments/steps of your solutions at space below each question or at extra A4 paper with question number and your name on the left-upper corner.

- You can write your answer with pencil or ball pen. Compasses and Rulers are allowed. Protractor, correction fluid, correction tapes, any electronic calculators and any electronic products of any kinds are not allowed.

記號　Notations

- $A \cup B$ 表示集合A與B之併集。
  $A \cup B$ represents the union of sets A and B.

- 符號$\lfloor x \rfloor$ 表示不大於實數x的最大整數。
  The symbol $\lfloor x \rfloor$ represents the largest integer which is smaller or equal to the real number x.

- $\sum_{1 \le i < j \le n} A_{ij} = A_{12} + \cdots + A_{1n} + A_{23} + A_{24} + \cdots + A_{2n} + A_{34} + \cdots + A_{n-1\ n}$.

- 三角形ABC的A—旁切圓是與三角形ABC的邊BC 以及邊AB、AC的延長線相切的圓。
  The A–excircle of triangle ABC is the circle that is tangent to the side BC.

- 三角形ABC的內心I是經過其三個頂點的三個內角平分線的唯一交點。
  The incenter I of triangle ABC is the unique intersection point of the three internal angle bisectors through its vertices.

- 三角形ABC 的外心O是其三邊垂直平分線的唯一交點。
  The circumcenter O of triangle ABC is the unique intersection point of the three perpendicular bisectors of its sides.

答案空格

1. (5分) 計算以下的和式
(5 marks) Evaluate the sum

$$\left\lfloor \sqrt{16} - \sqrt{6} \right\rfloor + \left\lfloor \sqrt{26} - \sqrt{16} \right\rfloor + \cdots + \left\lfloor \sqrt{2026} - \sqrt{2016} \right\rfloor \circ$$

A. 2 **B**. 4 **C**. 12 **D**. 24 **E**. 以上皆非 None of the above (1) [ ]

2. (5分) 設滿足 $b^3 = a^3 + 61$ 的整數對 (a, b) 之個數爲 N，則 $N =$

(5 marks) If N denotes the number of pairs (a, b) of integers such that $b^3 = a^3 + 61$, then $N =$

A. 0 **B**. 1 **C**. 2 **D**. 4 **E**. 以上皆非 None of the above (2) [ ]

3. (5分)考慮所有由9 個字母組成的字串，其中每個字母都是A或B。如果9個字母組成的字串不包含兩個連續的A 或三個連續的B，則稱該字串是「多樣化的」。例如，$ABABABABA$和$BBABABBAB$ 是多樣化的，但$AAABABBBB$ 和$BBABBAABB$ 則不是。問:有多少個不同的多樣化的字串?

(5 marks) Consider all 9–letter strings, where each letter is an A or a B. A 9–letter string is *diverse*, if it does not contain two consecutive A's nor three consecutive B's. For example, $ABABABABA$ and $BBABABBAB$ are diverse but $AAABABBBB$ and $BBABBAABB$ are not. How many diverse strings are there?

A. 10 **B**. 20 **C**. 30 **D**. 40 **E**. 以上皆非 None of the above (3)

4. (5分) 寫出一個二次多項式$p(x) = x^2 + ax + b$ 使得每個係數均爲整數且$p(\sqrt{2026} + \sqrt{2025}) = 0$。

(5 marks) Write down a quadratic polynomial $p(x) = x^2 + ax + b$ such that each coefficient is integer and $p(\sqrt{2026} + \sqrt{2025}) = 0$.

A. $x^2 + 90x + 1$ **B**. $x^2 + 90x - 1$ **C**. $x^2 - 90x + 1$
D. $x^2 - 90x - 1$ **E**. 以上皆非 None of the above (4)

5. (25 分) 考慮兩個不相交的有限正整數集 A 和 B，它們分別包含 n 個和 m 個元素。已知對於任意 $k \in A \cup B$，以下兩個條件中至少有一個成立：(i) $k+17 \in A$；(ii) $k-31 \in B$。證明：$17n = 31m$。

(25 marks) Consider two disjoint finite sets A and B of positive integers, having n and m elements respectively. It is known that for any $k \in A \cup B$, at least one of the following two conditions holds: (i) $k+17 \in A$; (ii) $k-31 \in B$. Prove that $17n = 31m$.

6. (25 分) 甲和乙兩人玩遊戲。遊戲開始時，甲在黑板上寫下一個正整數。然後他們輪流走步，乙先走。乙每走一步，都會將當前黑板上的數字 n 替換成 $n-a^2$，其中 a 爲正整數。甲每走一步，都會將當前黑板上的數字 m 替換成形如 m^k 的數字，其中 k 爲任意正整數。如果黑板上的數字變爲零，則乙獲勝。甲能否阻止乙勝出這遊戲嗎？

(25 marks) Amy and Bob play a game. At the beginning, Amy writes down a positive integer on a blackboard. Then they take turn to move, and Bob moves first. On each of his move, Bob replaces the current number n on the blackboard with a number of the form $n-a^2$, where a is positive integer. On each of her move, Amy replaces the current number m on the blackboard with a number of the form m^k, where k is any positive integer. Bob *wins* if the number on the blackboard becomes zero. Can Amy prevent Bob from winning the game?

7. (25分)設 n 爲正整數。在一個面上把$2n+1$ 枚硬幣排成一排，每枚硬幣要么是黑色，要么是白色。對每個硬幣，考慮它左側白色硬幣的數量及右側黑色硬幣的數量之和，稱這枚硬幣爲「好」，若這個和爲n。求「好」硬幣數量的奇偶性。

(25 marks) Let n be a positive integer. In a table, $2n+1$ coins are place in a row, each coin is either black or white in color. For each coin, consider the sum of the the number of white coins on the left plus the number of the black coins on the right, a coin is called *good*, if the sum is exactly n. Determine the parity of the number of good coins.

8. (25 分) 求式子 $S(x_1, x_2, x_3, x_4) = \sum_{1 \le i < j \le 4} (x_i + x_j)\sqrt{x_i x_j}$ 所能達到的最大值，其中 x_1, x_2, x_3, x_4 取遍所有的非負實數且滿足 $x_1 + x_2 + x_3 + x_4 = 1$，並寫出能使式子 S 達到最大值的 x_i $(1 \le i \le 4)$。

(25 marks) Find the maximum value of the expression $S(x_1, x_2, x_3, x_4) = \sum_{1 \le i < j \le 4} (x_i + x_j)\sqrt{x_i x_j}$ as x_1, x_2, x_3, x_4 run through all the non-negative real numbers with $x_1 + x_2 + x_3 + x_4 = 1$, and write down the values of x_i $(1 \le i \le 4)$ at which the maximum of S is achieved.

9. (25分) 設I、O、ω分別爲三角形ABC的內心、外心及外接圓。三角形ABC的A–旁切圓分別與直線AB、AC、BC相切於點K、L、M。若KL的中點位於ω上，求證I、M、O三點共線。

(25 marks) Let I, O, ω be the incenter, circumcenter, circumcircle of triangle ABC.
The A–excircle of ABC touches the straight lines AB, AC, BC at K, L, M respectively.
If the midpoint of the segment KL lies on the circle ω, prove that I, M, O are collinear.

